Event-Triggered Dissipative Tracking Control of Networked Control Systems With Distributed Communication Delay

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***Abstract*—This article presents the design of an event-trigger-based networked robust dissipative tracking control systems. An event-triggered mechanism (ETM) is proposed by introducing the probability density distribution of communication delay. Thus, it can yield less conservative results. The tracking error system is transferred by a distributed communication delay system wherein the delay probability density is viewed as the kernel of the distributed delay. The threshold of the proposed ETM is designed as a dynamic parameter to adapt the transmission of the control signal. By applying the Lyapunov method, sufficient conditions are achieved to ensure the stability of the tracking error system with strictly dissipative performance, which includes H_∞ performance and passive performance. Based on the dissipativity analysis condition, a co-design method with both tracking control and the ETM is derived in a united framework. Experiments on networked dc motor tracking system are provided to show the effectiveness of the presented method.**

***Index Terms*—Cyber-physical systems, dc motor tracking system, distributed communication delay, event-triggered mechanism.**

I. Introduction

With the deep integration of physical plants, computation, and communication and control technologies, cyber-physical systems (CPSs) have become a popular research topic owing to their wide applications in power grids, smart vehicles, and so on [1], [2]. With the increasing number of intelligent/control devices connected to a network, the quality of service of communication network with a tremendous amount of data exchange inevitably degrades due to limited communication resources. To save established network resources while maintaining a prescribed control performance, valid methods, including quantization [3]–[5] and the event-triggered mechanism (ETM) [6]–[11] have been investigated.

In the past few years, the ETM has been recognized as a preferable method of reducing redundant communication data owing to the feature that real communication data are updated only when the predesigned event-triggered condition is violated. Regarding this issue, the stability analysis of event-triggered control systems was derived in Girard [12]–[14] via the input-to-state stability (ISS) framework. Within this framework, an output event-triggered control strategy for CPSs with disturbance and measurement noise was presented in [15], where the ISS was ensured by a given observer and a controller. A codesign method with both controller and ETM parameters was proposed for linear networked systems in [16] by modeling the event-triggered control system as a time-delay system. Based on this strategy, extended results of CPSs were studied successively in [17]–[21]. To be specific, Gu *et al.* [17] investigated the event-triggered secure output control issue for continuous-time CPSs against cyber attacks. The controller and the ETM were codesigned by utilizing a Lyapunov technology. By using the ETM to mitigate the burden of both communication bandwidth and computation, and designing an observer to estimate the external disturbance, Liu *et al.* [18] investigated the networked tracking control problem for nonlinear multiagent systems. In [19], Shi *et al.* studied the event-triggered secure control issue for nonlinear CPSs with sensor saturation, where a Takagi–Sugeno (T–S) fuzzy model was used to model the nonlinear dynamic and a deception attack was considered. In [21], an H_∞ filter of nonlinear CPSs was derived based on a novel ETM by which it could reduce the spurious events caused by rapid changes arising from environment noises and external disturbances. It is noted that, in these results, the triggering threshold was designed as a constant, which facilitated the achievement of expected results. However, it is more reasonable to adjust the triggering threshold along with dynamic behavior.

Output tracking control has attracted much attention due to its wide applications in practical systems, such as robotic control [22], [23] and flight control [24], [25]. The goal of tracking control is to drive the system output to follow the given reference signal under a designed controller. Results on the output tracking control of CPSs can be found in [27]–[31] and

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the references therein. The research works in [27] studied the issue of output tracking control by considering communication delays in both the feedback and forward network channels. Output tracking control was investigated for CPSs with false data injection attacks and network-induced delays in [28]. Su *et al.* [29] investigated ETM-based tracking control for nonlinear systems with unknown Prandtl–Ishlinskii (PI) hysteresis. In [31], the tracking issue under a FlexRay communication protocol was developed in the framework of a hybrid model. It is noted that most of these results are based on an assumption that the communication network is ideal for transmitting all the sampled data among system components. However, network resources such as channel bandwidth and the energy used are limited. It is difficult to satisfy the above assumption in practical CPSs. To overcome the problem of nonideal communication in network-based tracking control, compared with the conventional communication mode, the ETM-based tracking control design is an alternative that can relieve the burden on the network by decreasing the amount of unnecessary data transmission.

Communication delay is a crucial issue that must be addressed the control design of CPSs. Improper control strategy would lead to the control performance degradation of the system with communication delay [26]. Most of the existing results, such as in [27] and [28], were obtained by using the upper and lower bounds of the communication delay. However, for real CPSs, such as Internet Protocol (IP)-based systems [32]–[35], communication delays are usually randomly distributed with some probability characteristics. From the perspective of system design, more available information on communication delays is conducive to obtaining less conservative results and better performance. In [36], an adaptive ETM was designed for tracking control, wherein the communication delays belonging to short and long delays were assumed to obey the Bernoulli distribution. Note that this approach uses the approximate cumulative probability of stochastic delays. In fact, the probability density information is more accurate for describing stochastic communication delays, which is rarely considered in existing results.

It is known that exogenous disturbances generally exist in real systems, which could deteriorate the system performance. To address this problem, a dissipative method has been extensively used for the analysis and synthesis of the system against disturbances. In [37], the problem of dissipative event-triggered control for networked systems was investigated by using a nonuniform sampling method. A decentralized event-triggered control was designed in [38] to ensure the dissipativity of the system with distributed multiple sensors. It is noteworthy that little attention has been devoted to investigating the dissipative event-triggered tracking control of CPSs with probability density information on communication delays.

Motivated by the above observations, this article is concerned with the design of the dissipative tracking control of CPSs with a dynamic ETM and distributed communication delays. The main contributions are as follows:

1) The model of networked tracking control system with probability density distribution (PDD) is established by introducing the probability density of network-induced communication delay. Compared to the existing literature on distributed delay for networked control systems, such as in [39], more statistical information on communication delay is used in the control design. Therefore, less conservative results could be obtained.

2) A dynamic triggering threshold regulated along with the states of the tracking system and reference system is introduced to the ETM, which is more practical than some ETMs with a constant triggering threshold parameter. Moreover, the PDD of communication delay is considered in the proposed ETM.

3) Sufficient conditions for the dissipativity of the event-triggered tracking system with consideration of PDD are developed. Compared with a system that does not consider the delay probability density such as the approach in [36], the proposed approach can lead to less conservative results. Simulations of a servo system are presented to demonstrate the effectiveness of the developed method.

The remainder of this article is organized as follows. In Section II, the addressed problem is formulated and in Section III, the analysis and design problem for the event-trigger-based networked robust dissipative tracking control is carried out. The validity of our proposed method is demonstrated through an example of networked dc motor in Section IV. Finally, we conclude this article in Section V.

Notation: In this article, $\natural\{X, Y\}$ stands for $Y^T XY$, i.e., $\natural\{X, Y\} \triangleq Y^T XY$. $\binom{a}{b}$ represents $\dfrac{a!}{(a-b)!b!}$. $\mathbf{He}(Y) = Y^T + Y$. The other notations are standard.

II. Preliminaries

Consider the following uncertain system:

$$\begin{cases} \dot{x}(t) = (A + \Delta A(t))x(t) + (B + \Delta B(t))u(t) + D\omega(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state vector, $\omega(t) \in \mathbb{R}^{n_\omega}$ is the exogenous disturbance in $\mathcal{L}_2[0\ \ \infty)$, $u(t) \in \mathbb{R}^{n_u}$ is the control input, and $y(t) \in \mathbb{R}^{n_y}$ is the system output. The reference model is considered as follows:

$$\begin{cases} \dot{x}_r(t) = Fx_r(t) + r(t) \\ y_r(t) = Hx_r(t) \end{cases} \tag{2}$$

where $r(t) \in \mathbb{R}^{n_r}$, $x_r(t) \in \mathbb{R}^{n_r}$, and $y_r(t) \in \mathbb{R}^{n_y}$ are the reference input, state and output, respectively. A, B, C, D, F, and H are known matrices with appropriate dimensions. $\Delta A(t)$ and $\Delta B(t)$ are norm-bounded uncertainties that satisfy

$$\begin{bmatrix} \Delta A(t) & \Delta B(t) \end{bmatrix} = E\Delta(t) \begin{bmatrix} N_1 & N_2 \end{bmatrix}$$

where E, N_1, and N_2 are constant matrices and $\Delta^T(t)\Delta(t) \leq I$.

In this article, a dynamic ETM will be developed to relieve the network burden, and output tracking controller will be designed to make the system output $y(t)$ in (1) follow the reference output $y_r(t)$ in (2) through a communication network.

First, we design the dynamic ETM given as below to determine the next triggering instant s_{k+1}

$$s_{k+1} = \sup_{s}\{s \geq s_k | e^T(s)\Psi e(s) < \delta(s)\xi^T(s_k)\Psi\xi(s_k)\} \tag{3}$$

where $\xi(s) = [\, x^T(s) \; x_r^T(s) \,]^T$, $s \in [s_k,\ s_{k+1})$. s_k is the latest triggering time. $e(s) = \xi(s) - \xi(s_k)$ means the error between the current signal $\xi(s)$ and the last triggered signal $\xi(s_k)$, Ψ is a positive weighting matrix that needs to be determined, and $\delta(s) \in [\delta_m,\ \delta_M]$ is a dynamic triggering threshold determined by the following adaptive rule:

$$\delta(s) = \delta_M - (\delta_M - \delta_m)e^{-\|\xi(s)\|},\ \ s \in [s_k,\ s_{k+1}). \qquad (4)$$

Remark 1: It is seen from (4) that the adaptive rule relies on the current state $\xi(s)$. This makes the dynamic ETM generate a larger threshold to reduce triggered signals when $\xi(s)$ approaches the equilibrium state, while a smaller threshold will be engendered when $\xi(s)$ is removed from the equilibrium state. Moreover, if one takes $\delta_M = \delta_m$, the adaptive law becomes $\delta(s) = \delta_M$ and the dynamic ETM is reduced to a static ETM with a constant threshold like in [8] and [40].

Remark 2: Based on the dynamic threshold, such as in [41] and [42] the threshold can also be regulated according to the system state. However, this may result in a large amount of computation resources, moreover, this adjustment is sometimes not as applicable as the method in (4).

When the triggering condition (3) is violated, a new signal $\xi(s_{k+1})$ will be sent to the controller side via the communication network. It is noteworthy that communication delay τ_{s_k} at the triggering time s_k with upper bound τ_M is usually randomly distributed.

The controller updates the information $K\xi(s_k)$ at the instant t_k, which is transmitted via the network at the triggering instant s_k. The state feedback tracking control law is constructed as follows:

$$u(t) = \mathcal{K}\xi(s_k) = K_1 x(s_k) + K_2 x_r(s_k),\ \ t \in [t_k,\ t_{k+1}) \qquad (5)$$

where $t_k = s_k + \tau_{s_k}$ and $t_{k+1} = s_{k+1} + \tau_{s_{k+1}}$, $\mathcal{K} \triangleq [K_1\ K_2]$ and K_1 and K_2 are the gains to be designed.

By applying a similar method in [46], we define the time-varying delay $\tau(t)$ as

$$\tau(t) = \frac{t_{k+1} - t}{t_{k+1} - t_k}\tau_{s_k} + \frac{t - t_k}{t_{k+1} - t_k}\tau_{s_{k+1}} \qquad (6)$$

which will be abbreviated to τ for the ease of derivation.

Combining (6) and triggering condition (3) implies that

$$e^T(t,\tau)\Psi e(t,\tau) < \delta(t-\tau)\xi^T(s_k)\Psi\xi(s_k) \qquad (7)$$

where $e(t,\tau) = \xi(t-\tau) - \xi(s_k)$, $t \in [t_k,\ t_{k+1})$, and $\tau \in [0,\ \tau_M]$.

It is noted that communication delay is usually subject to random distribution in a practical network environment. To consider this PDD, the kernel $\rho(\cdot)$ with $\int_0^{\tau_M} \rho(v)dv = 1$ was introduced to approximate the probability density function (PDF) of the communication delay, which is supposed to be known prior through statistical methods outlined in this article.

For the PDF $\rho(v)$, we define $\vartheta_0(v) \triangleq \rho(v)$ and the vector $\vartheta(v) = [\, \vartheta_0(v) \;\; \vartheta_1(v) \cdots \cdots \vartheta_\kappa(v) \,]^T$ with $\kappa \in \mathbb{N}$ and $v \in [-\tau_M,\ 0]$, where $\vartheta_i(v)$ and $\vartheta_j(v)$ for $i \neq j, i, j \in \{0, 1, \ldots, \kappa\}$ are linearly independent.

Then, for $\vartheta(v)$, we make the following assumption:

$$\frac{d\vartheta(v)}{dv} = \Theta\vartheta(v) \qquad (8)$$

where Θ is a matrix that belongs to $\Theta \in \mathbb{R}^{\kappa\times\kappa}$.

Based on the PDF of communication delay and condition (7), we can obtain the result in Lemma 1 as below.

Lemma 1: For condition (7), we can gain the following condition:

$$\varepsilon^T(t)\Psi\varepsilon(t) < \delta_M \natural \left\{ \Psi, \left(\int_{-\tau_M}^0 \rho(v)\xi(t+v)dv - \varepsilon(t) \right) \right\} \qquad (9)$$

where $\varepsilon(t) \triangleq \int_{-\tau_M}^0 \rho(v)(\xi(t+v) - \xi(s_k))dv$ and $\rho(v) \geq 0$.

Proof: By defining $v = -\tau$ and applying the Schur complement to (7), we have

$$\begin{bmatrix} -\delta(t+v)\xi^T(s_k)\Psi\xi(s_k) & e(t,v) \\ e^T(t,v) & -\Psi^{-1} \end{bmatrix} < 0. \qquad (10)$$

By multiplying (10) with $\rho(v)$ and integrating it over $[-\tau_M,\ 0]$, one can get

$$\int_{-\tau_M}^0 \rho(v) \begin{bmatrix} -\delta(t+v)\xi^T(s_k)\Psi\xi(s_k) & e(t,v) \\ e^T(t,v) & -\Psi^{-1} \end{bmatrix} dv < 0. \qquad (11)$$

By applying the Schur complement to (11) follows that:

$$\begin{aligned} &\natural \left\{ \Psi, \int_{-\tau_M}^0 \rho(v)e(t,v)dv \right\} \\ &< \int_{-\tau_M}^0 \rho(v)\delta(t-v)\xi^T(s_k)\Psi\xi(s_k)dv \\ &\leq \delta_M \xi^T(s_k)\Psi\xi(s_k) \end{aligned} \qquad (12)$$

which is equivalent to (9) with $\xi(s_k) = \int_{-\tau_M}^0 \rho(v)\xi(t+v)dv - \varepsilon(t)$. This completes the proof. ■

According to the definition of $\varepsilon(t)$ in Lemma 1, the control input in (5) can be rewritten as

$$u(t) = \mathcal{K}\xi(s_k) = \mathcal{K}\int_{-\tau_M}^0 \rho(v)\xi(t+v)dv - \mathcal{K}\varepsilon(t). \qquad (13)$$

By combining (8), one can gain

$$u(t) = \mathcal{K}\mathcal{I}\int_{-\tau_M}^0 \vartheta(v)\xi(t+v)dv - \mathcal{K}\varepsilon(t) \qquad (14)$$

from (13), where $\mathcal{I} = [\, I_{n_x+n_r} \;\; 0_{n_x+n_r,\kappa(n_x+n_r)} \,]$.

Then, the closed-loop system can be obtained as follows by substituting (14) into (1):

$$\begin{cases} \dot{\xi}(t) = (\mathcal{A}+\Delta\mathcal{A})\xi(t) + (\mathcal{B}+\Delta\mathcal{B})\mathcal{K}\mathcal{I}\int_{-\tau_M}^0 \vartheta(v)\xi(t+v)dv \\ \qquad\quad - (\mathcal{B}+\Delta\mathcal{B})\mathcal{K}\varepsilon(t) + \mathcal{D}\widetilde{\omega}(t) \\ z(t) = \mathcal{C}\xi(t) \end{cases} \qquad (15)$$

where $\widetilde{\omega}(t) = \begin{bmatrix}\omega^T(t) & r^T(t)\end{bmatrix}^T$,

$$\mathcal{A} = \begin{bmatrix} A & 0_{n_x,n_r} \\ 0_{n_r,n_x} & F \end{bmatrix},\ \Delta\mathcal{A} = \begin{bmatrix} \Delta A(t) & 0_{n_x,n_r} \\ 0_{n_r,n_x} & 0_{n_r} \end{bmatrix}$$

$$\mathcal{B} = \begin{bmatrix} B \\ 0_{n_r,n_u} \end{bmatrix},\ \Delta\mathcal{B} = \begin{bmatrix} \Delta B(t) \\ 0_{n_r,n_u} \end{bmatrix}$$

$$\mathcal{D} = \begin{bmatrix} D & 0_{n_x,n_\omega} \\ 0_{n_r,n_\omega} & I_{n_r} \end{bmatrix}, \mathcal{C} = \begin{bmatrix} C & -H \end{bmatrix}.$$

Remark 3: The PDF $\rho(v)$ is introduced in the closed-loop system (15), which can be viewed as a distributed input-delay system. More statistical information is introduced in the design process rather than only boundary information on the delay, by which less conservative results could be achieved.

Remark 4: From (9), one can see that the developed ETM has been converted into a new format with historical information. In fact, $\int_{-\tau_M}^{0} \rho(v)\xi(t+v)dv$ can be regarded as an expectation of the system state or average system state during the previous period.

The objective of this article is to design the controller in (5) and the weight matrix of ETM in (3) such that tracking error system (15) under these parameters is asymptotically stable with the dissipative performance, which is defined as follows.

Before proceeding further, the following definition and technical lemma will be introduced, which are helpful for obtaining the main results.

Definition 1 ([47]): For given matrices $M_1 = M_1^T$, M_2 and $M_3 = M_3^T$, the closed-loop system is strictly dissipative with the dissipativity performance bound $\gamma > 0$ if

$$\int_0^\infty z^T(t)M_1 z(t)dt + \int_0^\infty \tilde{\omega}^T(t)(M_3 - \gamma I)\tilde{\omega}(t)dt \geq 0$$
$$+ 2\int_0^\infty z^T(t)M_2\tilde{\omega}(t)dt \tag{16}$$

for $\tilde{\omega}(t) \in \mathcal{L}_2\ [0, +\infty)$, and $M_1 = -M_{11}^T M_{11}$,

$$M_2 = \begin{bmatrix} M_{21} \in \mathbb{R}^{n_y \times n_\omega} & M_{22} \in \mathbb{R}^{n_y \times n_r} \end{bmatrix}$$

$$M_3 = \begin{bmatrix} M_{31} \in \mathbb{R}^{n_\omega \times n_\omega} & M_{32}^T \in \mathbb{R}^{n_\omega \times n_r} \\ M_{32} \in \mathbb{R}^{n_r \times n_\omega} & M_{33} \in \mathbb{R}^{n_r \times n_r} \end{bmatrix}.$$

Remark 5: The following special cases can be got from the Definition 1.

1) H_∞ *Performance:* This can be gained from condition (16) by selecting $M_1 = -I$, $M_2 = 0$, and $M_3 = (\gamma^2 + \gamma)I$;

2) *Passive Performance:* This can be derived from condition (16) by choosing $M_1 = 0$, $M_2 = I$, and $M_3 = 2\gamma I$;

3) *Mixed H_∞ and Passive Performance:* This can be acquired by setting $M_1 = -\varphi I$, $M_2 = (1-\varphi)I$ and $M_3 = (\varphi(\gamma^2 - \gamma) + 2\gamma)I$ from condition (16), where $0 \leq \varphi \leq 1$ denotes the weighting scalar.

Lemma 2 ([48]): For The symmetric matrix $\mathfrak{R} > 0 \in \mathbb{R}^{n\times n}$, vector $x(\cdot) \in \mathbb{R}^n$, and $\vartheta(\cdot)$ defined in (8), we have

$$\int_a^b x^T(v)\mathfrak{R}x(v)dv \geq \natural\left\{(\mathfrak{W} \otimes \mathfrak{R}), \int_a^b \theta(v)x(v)dv\right\} \tag{17}$$

with $\mathfrak{W}^{-1} = \int_a^b \vartheta(v)\vartheta^T(v)dv > 0$, $\theta(v) \triangleq \vartheta(v) \otimes I_n$.

Remark 6: If we select the elements of vector $\vartheta(v)$ ($v \in [a,\ b]$) as Legendre polynomials

$$\vartheta_i(v) = \mathbb{L}_i\left(\frac{b-v}{b-a}\right)$$
$$= (-1)^i \sum_{j=0}^{i}(-1)^j \binom{i}{j}\binom{i+j}{j}\left(\frac{b-v}{b-a}\right)^j \tag{18}$$

with $\mathfrak{W}^{-1} = \mathrm{diag}\{b-a, \frac{b-a}{3}, \ldots, \frac{b-a}{2\kappa+1}\}$ for $i = 0, \ldots, \kappa$, which also ensures that (17) holds. Consequently, one can see that (17) in Lemma 2 is more general than the Bessel–Legendre inequality in [49].

III. Main Results

In this section, we first present sufficient conditions for the dissipative stability in Theorem 1, and then design the event-triggered tracking control for the CPSs in Theorem 2.

Theorem 1: For given scalars δ_M, τ_M, and β and matrices $\mathcal{K} = [K_1\ K_2]$, $M_1 = M_1^T \leq 0$, M_2, and $M_3 = M_3^T$, tracking error system (15) is asymptotically stable with a prescribed dissipative index γ if there exist symmetric matrices $\mathcal{P}$, $\mathcal{Q} > 0$, $\mathcal{R} > 0$, and $\Psi > 0$ and matrix $\mathcal{G}$ such that

$$\mathbb{P} > 0 \tag{19}$$

$$\Upsilon + \mathbf{He}(\mathbb{X}\mathbb{Y}) < 0 \tag{20}$$

where

$$\mathbb{P} = \mathcal{P} + \mathrm{diag}\{0_{n_x+n_r}, \mathfrak{W} \otimes \mathcal{Q}\}, \mathfrak{W}^{-1} = \int_{-\tau_M}^{0} \vartheta(v)\vartheta^T(v)dv$$

$$\Upsilon = \mathbf{He}(\mathbb{H}^T\mathcal{P}\mathbb{M}) + \delta_M \mathbb{I}_1^T\Psi\mathbb{I}_1 - \mathbf{He}(\mathbb{I}_2^T\mathcal{C}^T M_2 \mathbb{I}_3)$$
$$+ \mathrm{diag}\{0_{2n}, \Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4, \Upsilon_5\}$$

$$\Upsilon_1 = \mathcal{Q} + \tau_M\mathcal{R} - \mathcal{C}^T M_1\mathcal{C},\ \Upsilon_2 = -\mathcal{Q},\ \Upsilon_3 = -\mathfrak{W} \otimes \mathcal{R}$$

$$\Upsilon_4 = -\Psi,\ \Upsilon_5 = -(M_3 - \gamma I_{n_\omega+n_r})$$

$$\mathbb{I}_1 = \begin{bmatrix} 0_{n_x+n_r,3(n_x+n_r)} & \mathcal{I} & -I_{n_x+n_r} & 0_{n_x+n_r,n_\omega+n_r} \end{bmatrix}$$

$$\mathbb{I}_2 = \begin{bmatrix} I_{n_x+n_r} & 0_{n_x+n_r,(\kappa+4)(n_x+n_r)} & 0_{n_x+n_r,n_\omega+n_r} \end{bmatrix}$$

$$\mathbb{I}_3 = \begin{bmatrix} 0_{n_x+n_r,(\kappa+5)(n_x+n_r)} & I_{n_x+n_r,n_\omega+n_r} \end{bmatrix}$$

$$\mathbb{S} = \begin{bmatrix} \mathbb{S}_1 & \mathbb{S}_2 \end{bmatrix}, \mathbb{H} = \begin{bmatrix} \mathbb{H}_1 & \mathbb{H}_2 \end{bmatrix}, \widehat{\Theta} = \Theta \otimes I_{(\kappa+1)(n_x+n_r)}$$

$$\mathbb{S}_1 = \begin{bmatrix} I_{n_x+n_r} & 0_{n_x+n_r} & 0_{n_x+n_r} \\ 0_{\kappa(n_x+n_r),n_x+n_r} & \theta(0) & -\theta(-\tau_M) \end{bmatrix}$$

$$\mathbb{S}_2 = \begin{bmatrix} 0_{n_x+n_r,(\kappa+1)(n_x+n_r)} & 0_{n_x+n_r,n_x+n_r+n_\omega+n_r} \\ -\widehat{\Theta} & 0_{\kappa(n_x+n_r),n_x+n_r+n_\omega+n_r} \end{bmatrix}$$

$$\mathbb{H}_1 = \begin{bmatrix} 0_{n_x+n_r} & I_{n_x+n_r} & 0_{n_x+n_r} \\ 0_{\kappa(n_x+n_r),n_x+n_r} & 0_{\kappa(n_x+n_r),n_x+n_r} & 0_{\kappa(n_x+n_r),n_x+n_r} \end{bmatrix}$$

$$\mathbb{H}_2 = \begin{bmatrix} 0_{n_x+n_r,(n_x+n_r)(\kappa+1)} & 0_{n_x+n_r,n_x+n_r+n_\omega+n_r} \\ I_{(\kappa+1)(n_x+n_r)} & 0_{\kappa(n_x+n_r),n_x+n_r+n_\omega+n_r} \end{bmatrix}$$

$$\mathbb{X} = \begin{bmatrix} \mathcal{G}^T & \beta\mathcal{G}^T & 0_{n_x+n_r,(\kappa+3)(n_x+n_r)} & 0_{n_x+n_r,n_\omega+n_r} \end{bmatrix}^T$$

$$\mathbb{Y} = \begin{bmatrix} -I_{2n} \; \mathcal{A}+\Delta\mathcal{A} \; 0_{2n} \; (\mathcal{B}+\Delta\mathcal{B})\mathcal{K}\mathcal{I} \; -(\mathcal{B}+\Delta\mathcal{B})\mathcal{K} \; \mathcal{D} \end{bmatrix}.$$

Proof: See Appendix.

Theorem 2: For given parameters δ_M, τ_M, β, and ℓ and matrices $M_1 = M_1^T \leq 0$, M_2, and $M_3 = M_3^T$, the asymptotic stability and a prescribed dissipative index γ of the tracking error system (15) is ensured, if there exist symmetric matrices $\widehat{\mathcal{P}}$, $\widehat{\mathcal{Q}} > 0$, $\widehat{\mathcal{R}} > 0$, and $\widehat{\Psi} > 0$ and matrices $\mathcal{Z}$ and $\mathcal{J}$ such that

$$\widehat{\mathbb{P}} > 0$$

$$\begin{bmatrix} \widehat{\Upsilon} + \mathbf{He}(\widehat{\mathbb{X}}\widehat{\mathbb{Y}}) & \widehat{\Upsilon}_a & \ell\overline{\mathbb{X}}_1 & \overline{\mathbb{Y}}_1^T \\ * & -I & 0 & 0 \\ * & * & -\ell I & 0 \\ * & * & * & -\ell I \end{bmatrix} < 0 \tag{21}$$

where

$$\widehat{\mathbb{P}} = \widehat{\mathcal{P}} + \mathrm{diag}\{0_{n_x+n_r}, \mathcal{W}\otimes\widehat{\mathcal{Q}}\}$$

$$\widehat{\Upsilon} = \mathbf{He}(\mathbb{H}^T\widehat{\mathcal{P}}\mathbb{S}) + \delta_M\mathbb{I}_1^T\widehat{\Psi}\mathbb{I}_1 - \mathbf{He}(\mathbb{I}_2^T\mathcal{J}\mathcal{C}^T M_2\mathbb{I}_3)$$
$$+ \mathrm{diag}\left\{0_{2n}, \widehat{\Upsilon}_1, \widehat{\Upsilon}_2, \widehat{\Upsilon}_3, \widehat{\Upsilon}_4, \widehat{\Upsilon}_5\right\}$$

$$\widehat{\Upsilon}_1 = \widehat{\mathcal{Q}} + \tau_M\widehat{\mathcal{R}},\ \widehat{\Upsilon}_2 = -\widehat{\mathcal{Q}},\ \widehat{\Upsilon}_3 = -\mathcal{W}\otimes\widehat{\mathcal{R}}$$

$$\widehat{\Upsilon}_4 = -\widehat{\Psi},\ \widehat{\Upsilon}_5 = -(M_3 - \gamma I_{n_\omega+n_r})$$

$$\widehat{\mathbb{X}} = \begin{bmatrix} I_{n_x+n_r} \; \beta I_{n_x+n_r} \; 0_{n_x+n_r,(\kappa+3)(n_x+n_r)+n_\omega+n_r} \end{bmatrix}^T$$

$$\widehat{\mathbb{Y}} = \begin{bmatrix} -\mathcal{J}^T \; \mathcal{A}\mathcal{J}^T \; 0_{n_x+n_r} \; \mathcal{B}\mathcal{Z}\mathcal{I} \; -\mathcal{B}\mathcal{Z} \; \mathcal{D} \end{bmatrix}$$

$$\widehat{\Upsilon}_a^T = \begin{bmatrix} 0_{n_\omega+n_r,n_x+n_r} \; (\mathcal{J}M_{11}\mathcal{C})^T \; 0_{n_\omega+n_r,(\kappa+3)(n_x+n_r)+n_\omega+n_r} \end{bmatrix}^T$$

$$\overline{\mathbb{X}}_1 = \begin{bmatrix} \mathcal{E}^T \; \beta\mathcal{E}^T \; 0_{n_x+n_r,(\kappa+3)(n_x+n_r)+n_\omega+n_r} \end{bmatrix}^T$$

$$\overline{\mathbb{Y}}_1 = \begin{bmatrix} 0_{n_x,n_x+n_r} \; \mathcal{N}_1\mathcal{J}^T \; 0_{n_x,n_x+n_r} \; N_2\mathcal{Z}\mathcal{I} \; -N_2\mathcal{Z} \; 0_{n_x,n_\omega+n_r} \end{bmatrix}$$

$$\mathcal{E} = \begin{bmatrix} E \\ 0 \end{bmatrix},\ \mathcal{N}_1 = \begin{bmatrix} N_1 \; 0 \end{bmatrix}.$$

Moreover, the tracking controller gain can be derived by $\mathcal{K} = \begin{bmatrix} K_1 & K_2 \end{bmatrix} = \mathcal{Z}\mathcal{J}^{-T}$.

Proof: Consider the uncertainty in (1) that $\begin{bmatrix} \Delta A(t) & \Delta B(t) \end{bmatrix} = E\Delta(t)\begin{bmatrix} N_1 & N_2 \end{bmatrix}$, and (20) can be expressed as

$$\Theta + \mathbf{He}(\mathbb{X}\widetilde{\mathbb{Y}}) + \mathbf{He}(\overline{\mathbb{X}}\Delta(t)\overline{\mathbb{Y}}) < 0 \tag{22}$$

where

$$\mathbb{X} = \begin{bmatrix} \mathcal{G}^T & \beta\mathcal{G}^T & 0_{n_x+n_r,(\kappa+3)(n_x+n_r)} & 0_{n_x+n_r,n_\omega+n_r} \end{bmatrix}^T$$

$$\widetilde{\mathbb{Y}} = \begin{bmatrix} -I_{n_x+n_r} & \mathcal{A} & 0_{n_x+n_r} & \mathcal{B}\mathcal{K}\mathcal{I} & -\mathcal{B}\mathcal{K} & \mathcal{D} \end{bmatrix}$$

$$\overline{\mathbb{X}} = \begin{bmatrix} (\mathcal{G}\mathcal{E})^T & \beta(\mathcal{G}\mathcal{E})^T & 0_{n_x,(\kappa+3)(n_x+n_r)+n_\omega+n_r} \end{bmatrix}^T$$

$$\overline{\mathbb{Y}} = \begin{bmatrix} 0_{n_x,n_x+n_r} \; \mathcal{N}_1 \; 0_{n_x,n_x+n_r} \; N_2\mathcal{K}\mathcal{I} \; -N_2\mathcal{K} \; 0_{n_x,n_\omega} \end{bmatrix}.$$

By adopting [50, Lemma 2.2] to address the uncertain term in (22), one can gain

$$\Theta + \mathbf{He}(\mathbb{X}\widetilde{\mathbb{Y}}) + \ell\overline{\mathbb{X}}\,\overline{\mathbb{X}}^T + \ell^{-1}\overline{\mathbb{Y}}^T\overline{\mathbb{Y}} < 0 \tag{23}$$

where $\ell > 0$.

By pre- and postmultiplying $\mathbb{J} = \mathrm{diag}\{\mathcal{J}, \mathcal{J}, \mathcal{J}, I_{(n_x+n_r)(\kappa+1)}\otimes\mathcal{J}, \mathcal{J}, I\}$, $\mathcal{J} = \mathcal{G}^{-1}$, and its transpose $\mathbb{J}^T$ on both sides of (23), one can obtain

$$\mathbb{J}\Theta\mathbb{J}^T + \mathbf{He}(\widehat{\mathbb{X}}\widehat{\mathbb{Y}}) + \ell\overline{\mathbb{X}}_1\,\overline{\mathbb{X}}_1^T + \ell^{-1}\overline{\mathbb{Y}}_1^T\overline{\mathbb{Y}}_1 < 0$$

where

$$\widehat{\mathbb{X}} = \begin{bmatrix} I_{n_x+n_r} & \beta I_{n_x+n_r} & 0_{n_x+n_r,(\kappa+3)(n_x+n_r)+n_\omega+n_r} \end{bmatrix}^T$$

$$\widehat{\mathbb{Y}} = \begin{bmatrix} -\mathcal{J}^T & \mathcal{A}\mathcal{J}^T & 0_{n_x+n_r} & \mathcal{B}\mathcal{Z}\mathcal{I} & -\mathcal{B}\mathcal{Z} & \mathcal{D} \end{bmatrix}$$

$$\overline{\mathbb{X}}_1 = \begin{bmatrix} \mathcal{E}^T & \beta\mathcal{E}^T & 0_{n_x,(\kappa+1)(n_x+n_r)+n_\omega+n_r} \end{bmatrix}^T$$

$$\overline{\mathbb{Y}}_1 = \begin{bmatrix} 0_{n_x,n_x+n_r} \; \mathcal{N}_1\mathcal{J}^T \; 0_{n_x,n_x+n_r} \; N_2\mathcal{Z}\mathcal{I} \; -N_2\mathcal{Z} \; 0_{n_x,n_\omega} \end{bmatrix}.$$

Defining $\widehat{\mathcal{P}} = \mathcal{J}\mathcal{P}\mathcal{J}^T$, $\widehat{\mathcal{Q}} = \mathcal{J}\mathcal{Q}\mathcal{J}^T$, $\widehat{\mathcal{R}} = \mathcal{J}\mathcal{R}\mathcal{J}^T$, $\widehat{\Psi} = \mathcal{J}\Psi\mathcal{J}^T$, $\mathcal{J} = \mathcal{G}^{-1}$ and applying the Schur complement to (23) leads to

$$\begin{bmatrix} \widehat{\Theta} + \mathbf{He}(\widehat{\mathbb{X}}\widehat{\mathbb{Y}}) & \widehat{\Theta}_1 & \ell\overline{\mathbb{X}} & \overline{\mathbb{Y}}^T \\ * & -I & 0 & 0 \\ * & * & -\ell I & 0 \\ * & * & * & -\ell I \end{bmatrix} < 0 \tag{24}$$

which is equivalent to (21). This completes the proof. ■

IV. Numerical Example

In this section, we develop a software-in-the-loop simulation platform to manifest the effectiveness of the proposed method. DC motors are used in many industrial fields for theirs convenience of control design and their control strategies have been an important research topic [43], [44]. From Fig. 1, one can see that the platform consists of a PC and a wireless apparatus with a sender and receiver developed by the Arduino and Zigbee modules. The modules of the dc motor, the controller, and the ETM are implemented in MATLAB/Simulink on the PC, as shown in the dotted box on the right side of Fig. 1. The motor system dynamics are captured by the following differential

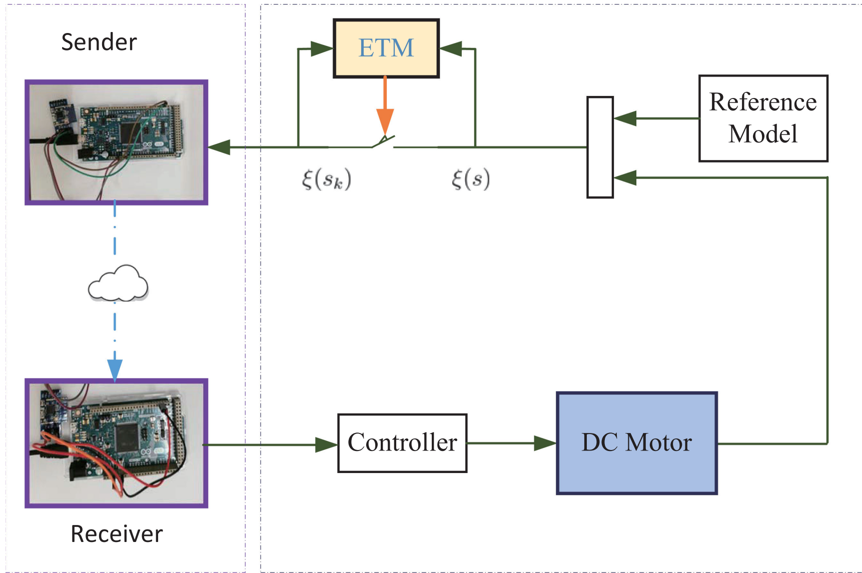

Fig. 1. Simulation platform of networked dc motor tracking system.

TABLE I
NOMENCLATURE OF DC MOTOR SYSTEM

Symbol	Quantity
$V_s(t)$	the armature voltage
$i_s(t)$	the armature current
$\varpi(t)$	the angular velocity
K_e	the counter electromotive force constant
K_m	the motor torque constant
R_s	the resistance of the armature winding
L_s	the inductance of the armature winding
J_s	the inertia of the motor and load
B_s	the viscous friction coefficient of the motor

equations [45]:

$$\begin{cases} \dfrac{di_s(t)}{dt} = -\dfrac{R_s}{L_s}i_s(t) - \dfrac{K_e}{L_s}\varpi(t) + \dfrac{1}{L_s}V_s(t) + \omega(t) \\ \dfrac{d\varpi(t)}{dt} = -\dfrac{B_s}{J_s}\varpi(t) + \dfrac{K_m}{J_s}i_s(t) \\ y(t) = v(t) = \dfrac{30}{\pi}\varpi(t) \end{cases} \tag{25}$$

where symbols of the dc motor system are listed in Table I.

Let $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} i_a(t) \\ \varpi(t) \end{bmatrix}$, $u(t) = V_s(t)$ and $y(t) = v(t)$, where $v(t)$ means the motor speed. $x(t)$, $u(t)$, and $y(t)$ are state, input, and output of the dc motor, respectively.

Then, we have the following system parameters:

$$A = \begin{bmatrix} -\dfrac{R_s}{L_s} & -\dfrac{K_e}{L_s} \\ \dfrac{K_m}{J_s} & -\dfrac{B_s}{J_s} \end{bmatrix}, B = \begin{bmatrix} \dfrac{1}{L_s} \\ 0 \end{bmatrix}, C = \begin{bmatrix} 0 & \dfrac{30}{\pi} \end{bmatrix}, D = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

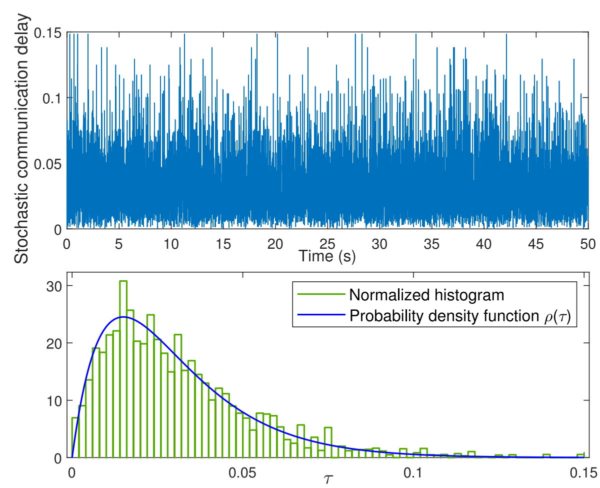

Fig. 2. Stochastic communication delay and its PDF $\rho(\tau)$.

with $R_s = 0.965(\Omega)$, $L_s = 0.0022(H)$, $K_e = 0.1201(\mathrm{V{\cdot}s/rad})$, $K_m = 0.1201(N{\cdot}\mathrm{m/A})$, $B_s = 0.1296(\mathrm{N/ms})$ and $J_s = 0.1182(\mathrm{kg{\cdot}m^2})$.

The reference model is considered as follows:

$$\begin{cases} \dot{x}_r(t) = -x_r(t) + r(t) \\ y_r(t) = 0.5x_r(t). \end{cases} \tag{26}$$

The uncertainties in (25) are considered as $\begin{bmatrix} \Delta A(t) & \Delta B(t) \end{bmatrix} = E\sin(t)I_2 \begin{bmatrix} N_1 & N_2 \end{bmatrix}$ with $E = 0.3I_2$, $N_1 = 0.1I_2$, and $N_2 = [0.2\ 0.1]^T$.

In this example, from some existing results [32], [49], the distributed communication delay with the upper bound $\tau_M = 0.15$ satisfying the Gamma distribution is considered. The delay distribution histogram is normalized in Fig. 2. In this example, the PDF for $\tau \in [0, \tau_M]$ is assumed to be

$$\rho(\tau) = 4445\tau e^{-67\tau}$$

where $\int_0^{\tau_M} \rho(\tau)d\tau = 1$ and the curve of $\rho(\tau)$ is also displayed in Fig. 2. $\vartheta_1(\tau)$ is chosen to be $4445\mathrm{e}^{-67\tau}$ to satisfy (8) for $\tau \in [0, \tau_M]$. Then, we have

$$\Theta = \begin{bmatrix} 67 & 1 \\ 0 & 67 \end{bmatrix}, \mathfrak{W} = \left(\int_{-\tau_M}^{0} \vartheta(v)\vartheta^T(v)dv \right)^{-1}$$

with $\vartheta(v) = \begin{bmatrix} -4445v\mathrm{e}^{67v} \\ -4445\mathrm{e}^{67v} \end{bmatrix}$.

Next, we discuss the following two cases by using the proposed method: the dissipative tracking control and the H_∞ tracking control.

Case 1 Dissipative Tracking Control: By setting $M_{11} = 0.55$, $M_{21} = 0.2$, $M_{22} = -0.2$, $M_{31} = 1$, $M_{32} = 0$, $M_{33} = 1$, $\gamma = 0.2$, $\delta_M = 0.25$, $\delta_m = 0.001$, $\beta = 30$, $\ell = 0.5$, the tracking controller and triggering weight matrix are solved by Theorem 2 as follows:

$$\mathcal{K} = \begin{bmatrix} -0.0189 & -5.7669 & 0.6259 \end{bmatrix}$$

$$\Phi = \begin{bmatrix} 0.0013 & 0.2161 & -0.0234 \\ 0.2161 & 65.7748 & -7.1356 \\ -0.0234 & -7.1356 & 0.7748 \end{bmatrix}. \tag{27}$$

Case 2 H_∞ Tracking Control: By setting $M_{11} = 1$, $M_{21} = 0$, $M_{22} = 0$, $M_{31} = \gamma^2 + \gamma$, $M_{32} = 0$, $M_{33} = \gamma^2 + \gamma$, $\delta_M = 0.25$, $\delta_m = 0.001$, $\beta = 5$, and $\ell = 0.5$, the optimal H_∞ tracking performance obtained by Theorem 2 is $\gamma = 0.2336$, and the tracking controller and triggering weight matrix are solved by Theorem 2 as follows:

$$\mathcal{K} = \begin{bmatrix} -0.0205 & -6.4524 & 0.7125 \end{bmatrix}$$

$$\Phi = \begin{bmatrix} 0.0029 & 0.8984 & -0.0992 \\ 0.8984 & 283.4278 & -31.2979 \\ -0.0992 & -31.2979 & 3.4561 \end{bmatrix}. \tag{28}$$

In the later simulation, we choose $\omega(t) = 12\cos(t)e^{-0.1t}$ and the initial conditions are taken as $x(0) = [-0.5\ 0.3]^T$ and $x_r(0) = 0.3$. The reference input is given by

$$r(t) = \begin{cases} 1, & 3 \leq t \leq 12 \\ 0.5, & 12 < t \leq 20 \\ 0, & \text{otherwise} \end{cases}$$

which is shown in Fig. 3.

For the dissipative tracking control case (Case 1), by using the parameters of the controller and the ETM in (27), one can get the output trajectories of the dc motor and the reference model, as depicted in Fig. 3. Accordingly, the release time instants and the triggering threshold are shown in Fig. 4. For H_∞ tracking control case (Case 2), under the same reference input, the responses of the dc motor output and the reference trajectory are produced in Fig. 5 by using the parameters in (28). From these figures, it is observed that the reference output $y_r(t)$ is tracked effectively by the system outputs $y(t)$ under both the dissipative tracking

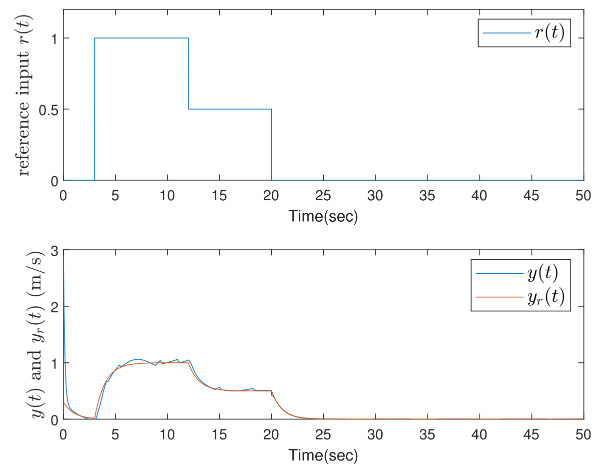

Fig. 3. Reference input $r(t)$ in (26), the outputs $y(t)$ in (25) and $y_r(t)$ in (26) under Case 1.

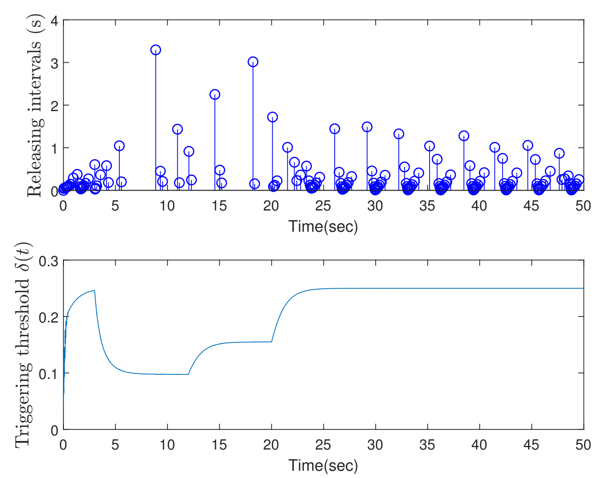

Fig. 4. Releasing intervals and the triggering threshold $\delta(t)$ under Case 1.

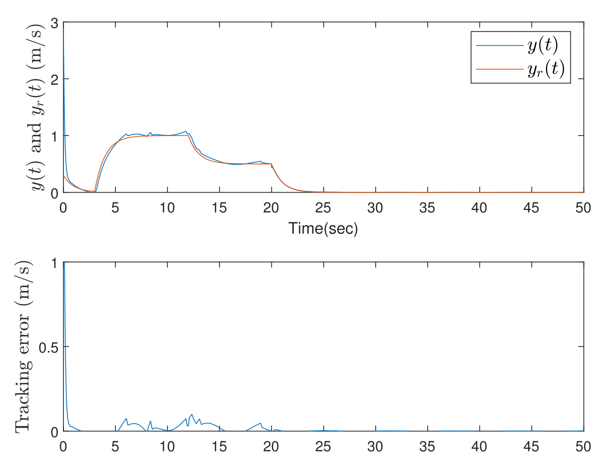

Fig. 5. Outputs $y(t)$ in (25) and $y_r(t)$ in (26) and the tracking error under Case 2.

TABLE II
COMPARISON OF THE H_∞ PERFORMANCE INDEX γ

Methods	γ
Method in [36] with $\tau_1 = \tau_M$, $\varrho_1 = 1$	0.7746
Method in [36] with $\tau_1 = 0.05$, $\varrho_1 = 0.84$	0.6547
Method in [36] with $\tau_1 = 0.025$, $\varrho_1 = 0.49$	0.7623
Theorem 2	0.2336

TABLE III
COMPARISON RESULTS OF NOV IN $\mathcal{P}$

Methods	NoV
Legendre polynomials with $\kappa = 5$	231
Legendre polynomials with $\kappa = 3$	120
Our method with $\kappa = 1$	45

control case and H_∞ tracking control case. Meanwhile, one can calculate the average triggering interval over the time interval $[0, 50\text{ s}]$ is 0.2272. It is noticed that the releasing instant is decided by the proposed adaptive ETM. From Fig. 4, one can see that the threshold of ETM is time-varying, which is depended on the system state rather than a fixed constant. By using such an ETM, a lower amount of data transmission is needed to guarantee the tracking performance.

To show the effectiveness of our proposed method of using the PDF to characterize communication delay, a comparison of the results from two aspects with those of existing methods are provided: 1) Divide the communication delay interval into two parts: short delay ($\tau \in [0, \tau_1)$ and long delay ($\tau \in [\tau_1, \tau_M)$, such as in [36]. The occurring probability in each interval is different; 2) approximate the PDF by using Legendre polynomials, such as in [49].

The comparison of the optimal H_∞ performance between our method and that in [36] is shown in Table II. From Table II, it is shown that the smaller optimal H_∞ index γ can be obtained by our method, which means that the controller design with our proposed PDD-based communication delay could lead to less conservative results.

As for the approximation method, [49] utilized Legendre polynomials to approximate the probability density $\rho(\tau)$. For a given κ, $\hat{\rho}(\tau)$ is expressed as

$$\hat{\rho}(\tau) = \sum_{i=0}^{\kappa} \frac{2i+1}{\tau_M} \int_0^{\tau_M} \rho(s)\mathbb{L}_i\left(\frac{\tau_M - s}{\tau_M}\right) ds$$

where $\mathbb{L}_i(\frac{\tau_M-s}{\tau_M}) = (-1)^i \sum_{j=0}^{i} (-1)^j \binom{i}{j}\binom{i+j}{j}\left(\frac{\tau_M - s}{\tau_M}\right)^j$. The approximation function $\hat{\rho}(\tau)$ with different degrees is drawn in Fig. 6, from which one can see that as the degree κ of Legendre polynomials increases, the approximation error $\rho(\tau) - \hat{\rho}(\tau)$ decreases. Meanwhile, the numbers of decision variables (NoV) in $\mathcal{P}$ in the Legendre polynomials method and our proposed method are listed in Table III. For example, when we choose $\kappa = 5$, $\rho(\tau)$ is approximated by $\hat{\rho}(\tau)$ and the

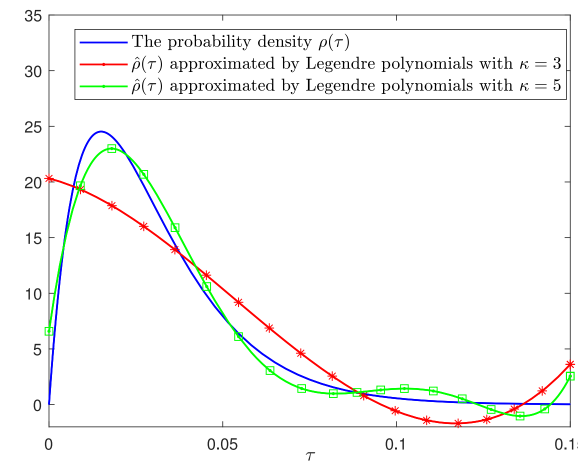

Fig. 6. $\rho(\tau)$ and $\hat{\rho}(\tau)$ with $\kappa = 3$ and $\kappa = 5$.

NoV in $\mathcal{P}$ is 231, which is much greater than NoV$=45$ in our method with $\kappa = 1$. This demonstrates that our approach can utilize the PDF $\rho(\tau)$ directly without approximation error and requires fewer decision variables than the approach [49] based on Legendre polynomials.

V. CONCLUSION

In this article, we have developed the dissipative output tracking control of CPSs with PDD-based communication delay and a dynamic ETM. A new model of distributed communication delay has been established by introducing a PDF method. For the convenience of deriving PDF-based delay-dependent results, we reconstructed the ETM and the tracking error system. The designed triggering threshold of the PDF-based ETM is adjusted by the system state, thereby dynamically changing the data-releasing rate. Based on the proposed model, a codesign method for both the tracking controller and the triggering weighting matrix has been presented. Since more statistical information is used in the derivation, less conservative results can be obtained. Finally, a numerical example of a servo tracking system is executed to show the effectiveness of our proposed method.

APPENDIX
PROOF OF THEOREM 1

The Lyapunov–Krasovskii functional is chosen

$$V(t) = V_1(t) + V_2(t) \tag{29}$$

where

$$V_1(t) = \zeta^T(t)\mathcal{P}\zeta(t)$$
$$V_2(t) = \int_{-\tau_M}^{0} \xi^T(t+v)[\mathcal{Q} + (v+\tau_M)\mathcal{R}]\xi(t+v)dv$$

with $\zeta(t) = \left[\begin{smallmatrix}\xi(t)\\ \int_{-\tau_M}^0 \theta(v)\xi(t+v)dv\end{smallmatrix}\right]$.

Applying Lemma 2 yields

$$\int_{-\tau_M}^{0} \natural\{\mathcal{Q}, \xi(t+v)\}\, dv$$

$$\geq \natural\left\{(\mathfrak{W}\otimes\mathcal{Q}), \int_{-\tau_M}^{0}\theta(v)\xi(t+v)\right\}dv \tag{30}$$

with $\mathfrak{W}=(\int_{-\tau_M}^{0}\vartheta(v)\vartheta^T(v)dv)^{-1}$ and $\theta(v)=\vartheta(v)\otimes I_{n_x+n_r}$.

From (29) and (30), one has

$$V(t)\geq \zeta^T(t)\mathbb{P}\zeta(t)+\int_{-\tau_M}^{0}\natural\{(v+\tau_M)\mathcal{R},\xi(t+v)\}\,dv \tag{31}$$

where $\mathbb{P}=\mathcal{P}+\mathrm{diag}\{0_{2n},\mathfrak{W}\otimes\mathcal{Q}\}$.

Thus, the Lyapunov–Krasovskii functional $V(t)$ with the conditions $\mathcal{Q}>0$, $\mathcal{R}>0$, and $\mathbb{P}>0$ is guaranteed to be positive.

Next, one can compute the derivative of $V(t)$ along the solution of system (15) as follows:

$$\dot{V}_1(t)=2\zeta^T(t)\mathcal{P}\dot{\zeta}(t) \tag{32}$$

$$\begin{aligned}\dot{V}_2(t)&=\xi^T(t)(\mathcal{Q}+\tau_M\mathcal{R})\xi(t)-\xi^T(t-\tau_M)\mathcal{Q}\xi(t-\tau_M)\\&\quad-\int_{-\tau_M}^{0}\xi^T(t+v)\mathcal{R}\xi(t+v)dv.\end{aligned} \tag{33}$$

From (8), one has

$$\begin{aligned}\frac{d}{dt}\int_{-\tau_M}^{0}\theta(v)\xi(t+v)dv&=\theta(0)\xi(t)-\theta(-\tau_M)\xi(t-\tau_M)\\&\quad-\widehat{\Theta}\int_{-\tau_M}^{0}\theta(v)\xi(t+v)dv.\end{aligned} \tag{34}$$

For descriptive convenience, we define

$$\begin{aligned}\chi^T(t)=\big[&\dot{\xi}^T(t)\ \xi^T(t)\ \xi^T(t-\tau_M)\\&\left(\textstyle\int_{-\tau_M}^{0}\theta(v)\xi(t+v)dv\right)^T\ \varepsilon^T(t)\ \tilde{\omega}^T(t)\Big].\end{aligned}$$

Then, from (34), one has

$$\zeta(t)=\mathbb{H}\chi(t),\ \dot{\zeta}(t)=\mathbb{S}\chi(t) \tag{35}$$

where $\mathbb{H}$ and $\mathbb{S}$ are given in Theorem 1.

Recalling Definition 1, we denote

$$\begin{aligned}\mathscr{J}(t)&=\dot{V}(t)-z^T(t)M_1z(t)\\&\quad-2z^T(t)M_2\tilde{\omega}(t)-\tilde{\omega}^T(t)(M_3-\gamma I)\tilde{\omega}(t).\end{aligned}$$

It follows that

$$\begin{aligned}\mathscr{J}(t)&=2\xi^T(t)\mathbb{H}^T\mathcal{P}\mathbb{S}\xi(t)+\xi^T(t)(\mathcal{Q}+\tau_M\mathcal{R})\xi(t)\\&\quad-\xi^T(t-\tau_M)\mathcal{Q}\xi(t-\tau_M)-z^T(t)M_1z(t)\\&\quad-\mathbf{He}(z^T(t)M_2\tilde{\omega}(t))-\tilde{\omega}^T(t)(M_3-\gamma I)\tilde{\omega}(t)\\&\quad-\int_{-\tau_M}^{0}\xi^T(t+\tau)\mathcal{R}\xi(t+\tau)d\tau.\end{aligned} \tag{36}$$

Lemma 1 yields

$$\varepsilon^T(t)\Psi\varepsilon(t)<\delta_M\natural\left\{\Psi,\left(\int_{-\tau_M}^{0}\rho(v)\xi(t+v)dv-\varepsilon(t)\right)\right\}. \tag{37}$$

Due to

$$\int_{-\tau_M}^{0}\rho(v)\xi(t+v)dv=\mathcal{I}\int_{-\tau_M}^{0}\theta(v)\xi(t+v)dv \tag{38}$$

one can obtain

$$\varepsilon^T(t)\Psi\varepsilon(t)<\delta_M\chi^T(t)\mathbb{I}_1^T\Psi\mathbb{I}_1\chi(t) \tag{39}$$

from (37).

In light of (36) and (39), one has

$$\begin{aligned}\mathscr{J}(t)&\leq 2\chi^T(t)\mathbb{H}^T\mathbb{P}\mathbb{S}\chi(t)+\xi^T(t)(\mathcal{Q}+\tau_M\mathcal{R})\xi(t)\\&\quad-\xi^T(t-\tau_M)\mathcal{Q}\xi(t-\tau_M)-z^T(t)M_1z(t)\\&\quad-\mathbf{He}(z^T(t)M_2\tilde{\omega}(t))-\tilde{\omega}^T(t)(M_3-\gamma I)\tilde{\omega}(t)\\&\quad+\delta_M\chi^T(t)\mathbb{I}_1^T\Psi\mathbb{I}_1\chi(t)-\varepsilon^T(t)\Psi\varepsilon(t)\\&\quad-\int_{-\tau_M}^{0}\natural\{\mathcal{R},\xi(t+v)dv\}.\end{aligned} \tag{40}$$

Adopting Lemma 2 to address the integral term $-\int_{-\tau_M}^{0}\natural\{\mathcal{R},\xi(t+v)dv\}$ results in

$$\begin{aligned}&-\int_{-\tau_M}^{0}\natural\{\mathcal{R},\xi(t+v)dv\}\\&\leq-\natural\left\{(\mathfrak{W}\otimes\mathcal{R}),\int_{-\tau_M}^{0}\theta(v)\xi(t+v)dv\right\}.\end{aligned} \tag{41}$$

By combining (40) and (41), it follows that:

$$\mathscr{J}(t)\leq\chi^T(t)\Upsilon\chi(t). \tag{42}$$

Clearly, (43) is a sufficient condition for ensuring $\mathscr{J}(t)<0$, which guarantees the dissipative stability of system (15)

$$\chi^T(t)\Upsilon\chi(t)<0. \tag{43}$$

From (38) and (15), one knows that

$$\mathbb{Y}\chi(t)=0. \tag{44}$$

Combining (43) results in

$$\mathscr{J}(t)\leq\chi^T(t)\left(\Upsilon+\mathbf{He}\left(\mathbb{X}\mathbb{Y}\right)\right)\chi(t)<0 \tag{45}$$

which is ensured by (20).

The dissipative stability of system (15) is guaranteed, according to Definition 1, by integrating the index $\mathscr{J}(t)$ over the time interval $[0,\infty)$. ■

Remark 7: A relaxed constraint on these variables is given in Theorem 1, in which matrix $\mathcal{P}$ is not required to be positive. Compared with most of the existing results, which require all the Lyapunov items to be negative, less conservative results can be obtained by using Theorem 1.

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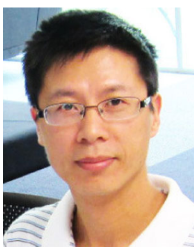

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